

GED Math Graphics: Illustrated Guide to Self-Confidence

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Unit 3

BASIC GEOMETRY

- Angles
- Triangles
- Right Triangles
- Similar Triangles
- Perimeter, Area and Volume

by Howard Myers, Ed.D.

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Similar Triangles

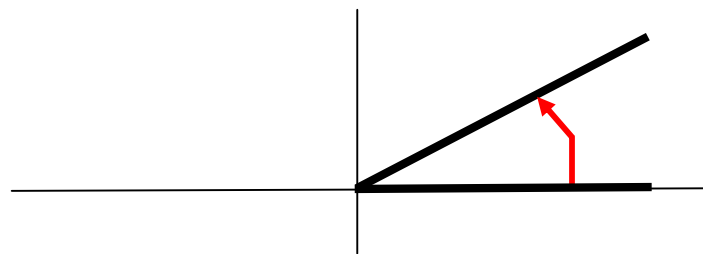
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Perimeter, Area and Volume

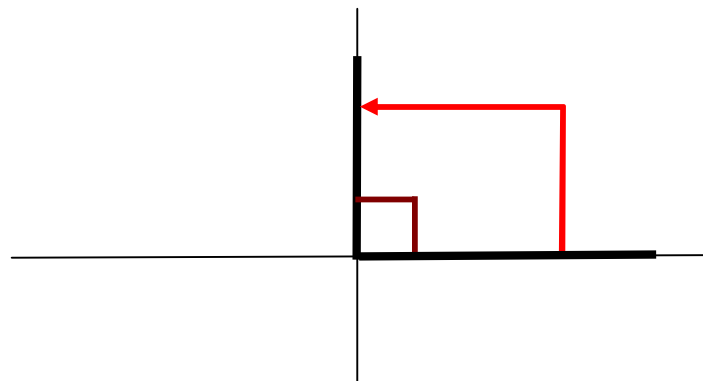
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*** Typical GED Questions

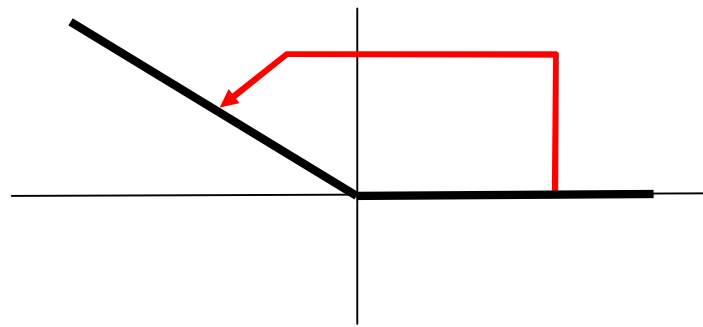
Important Angles



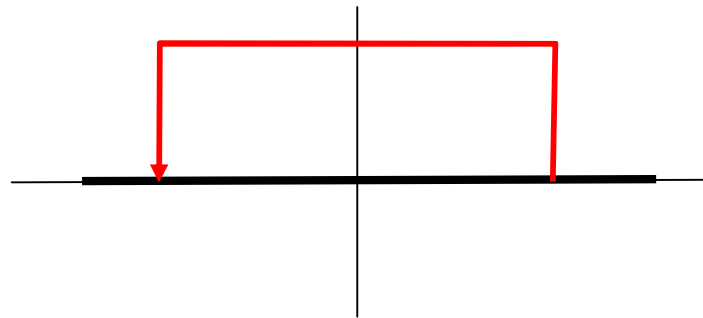
Acute
Less than 90°



Right
Equal to 90°

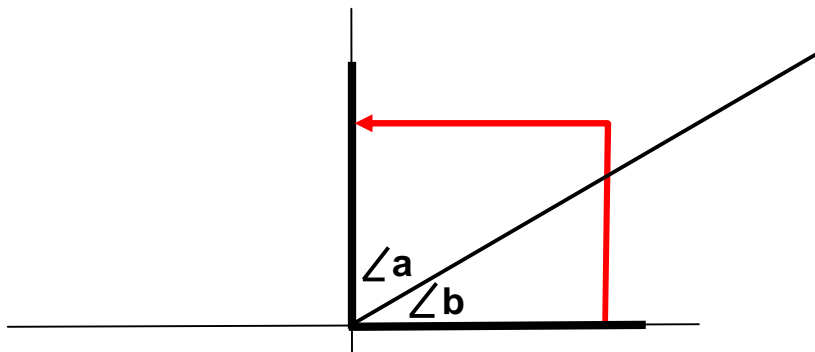


Obtuse
Greater than 90° , Less than 180°



Straight
Equal to 180°

Complementary Angles



Rule: $\angle a + \angle b = 90^\circ$

Typical GED Question

If $\angle a$ measures 29.3° , what is the measure of $\angle b$?

Solution:

Step 1

$$29.3^\circ + \angle b = 90^\circ$$

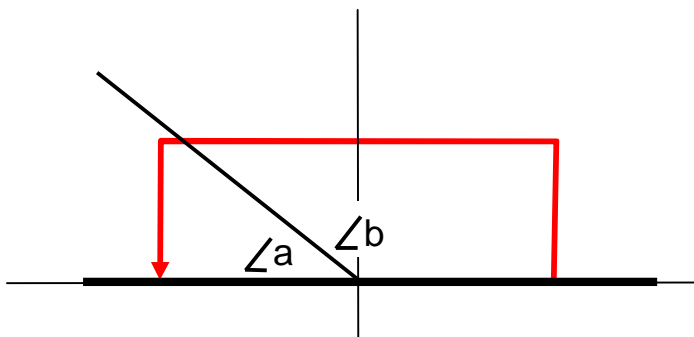
Step 2

$$\begin{array}{r} 90.0^\circ \\ -29.3^\circ \\ \hline 60.7^\circ \end{array} \text{ Answer}$$

Check:

$$29.3^\circ + 60.7^\circ = 90^\circ$$

Supplementary Angles



Rule: $\angle a + \angle b = 180^\circ$

Typical GED Question

If $\angle a$ measures 43.7° , what is the measure of $\angle b$?

Solution:

Step 1

$$43.7^\circ + \angle b = 180^\circ$$

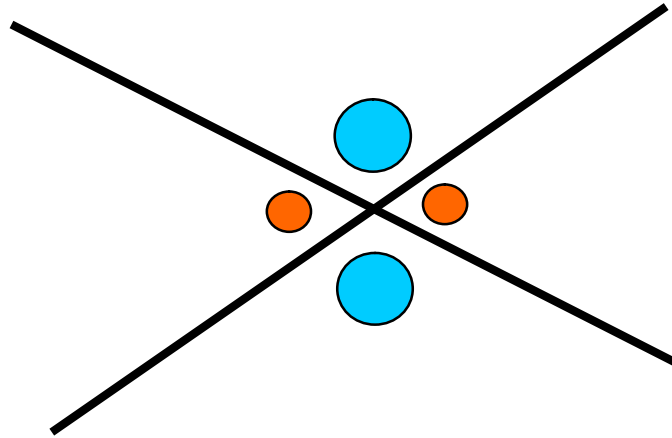
Step 2

$$\begin{array}{r} 180.0^\circ \\ -43.7^\circ \\ \hline 136.3^\circ \end{array} \text{ Answer}$$

Check:

$$43.7^\circ + 136.3^\circ = 180^\circ$$

When Two Straight Lines Intersect



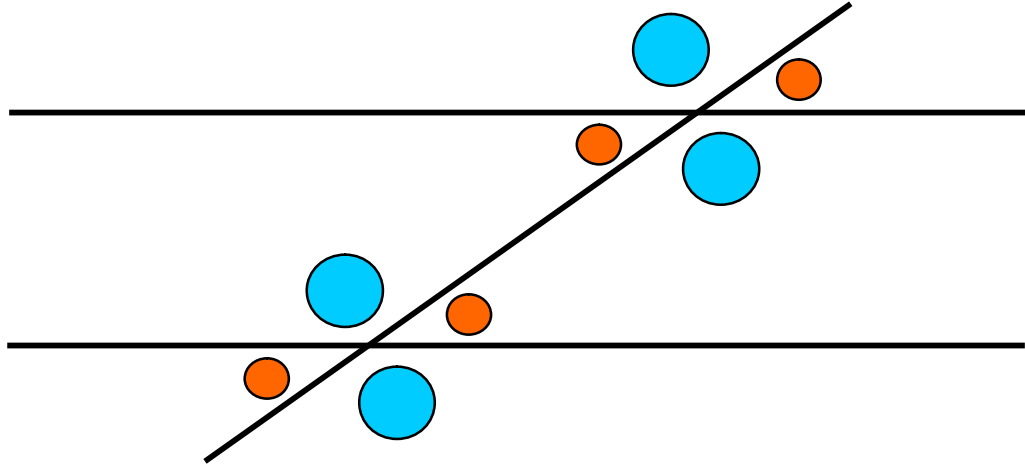
You Always Know:

$$\angle \text{blue} = \angle \text{blue}$$

$$\angle \text{orange} = \angle \text{orange}$$

$$\angle \text{blue} + \angle \text{orange} = 180^\circ$$

Parallel Lines with Angles



You Always Know:

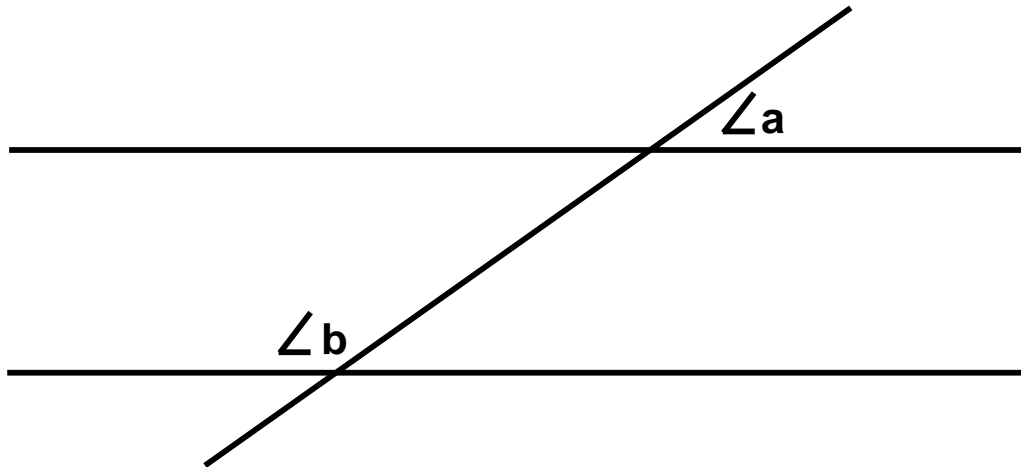
$$\angle \text{blue} = \angle \text{blue} = \angle \text{blue} = \angle \text{blue}$$

$$\angle \text{orange} = \angle \text{orange} = \angle \text{orange} = \angle \text{orange}$$

$$\angle \text{blue} + \angle \text{orange} = 180^\circ$$

Parallel Lines with Angles

Typical GED Question



If $\angle a$ measures 32° , what is the measure of $\angle b$?

Rule: $\angle \text{blue circle} + \angle \text{orange circle} = 180^\circ$

So: $\angle a + \angle b = 180^\circ$

Solution:

Step 1

$$32^\circ + \angle b = 180^\circ$$

Step 2

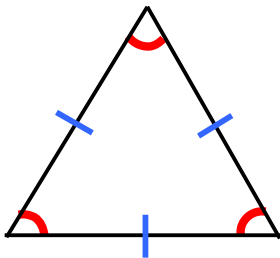
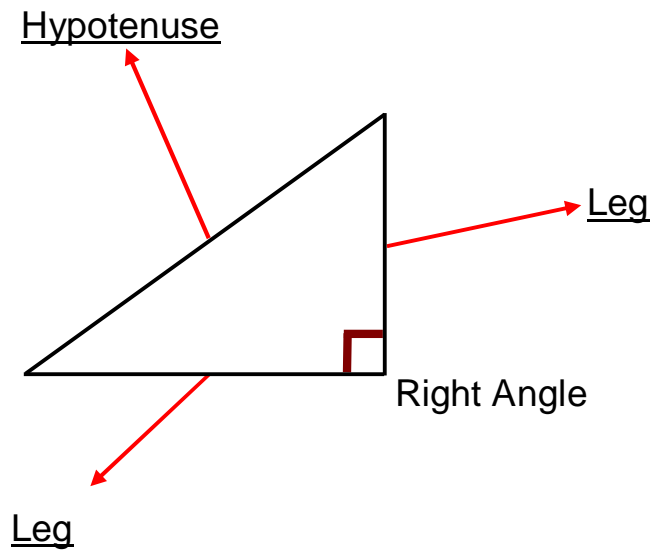
$$\begin{array}{r} 180^\circ \\ -32^\circ \\ \hline 148^\circ \end{array} \text{ Answer}$$

Check:

$$32^\circ + 148^\circ = 180^\circ$$

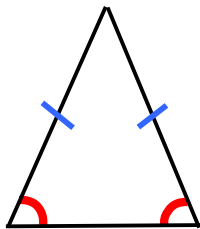
Special Triangles

Right Triangle



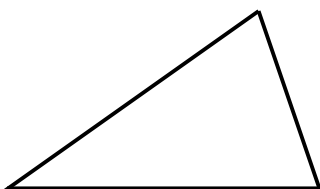
Equilateral

- 3 equal sides, 3 equal angles



Isoceles

- 2 equal sides, 2 equal angles



Scalene

- All sides and angles unequal

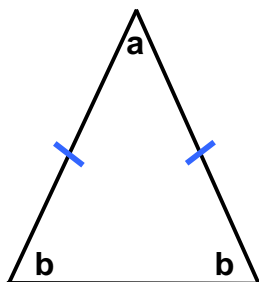
Find the Missing Angle

Typical GED Questions

Isoceles

Rule: $\angle a + \angle b + \angle b = 180^\circ$

If $\angle b = 65^\circ$, Then $\angle a = ???$



Solution

$$1. \angle a + \underbrace{65^\circ + 65^\circ}_{130^\circ} = 180^\circ$$

$$2. \angle a + 130^\circ = 180^\circ$$

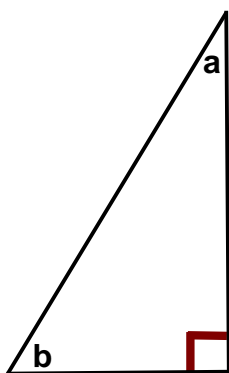
$$3. \begin{array}{r} 180^\circ \\ - 130^\circ \\ \hline \angle a = 50^\circ \end{array}$$

Check: $50^\circ + 65^\circ + 65^\circ = 180^\circ$

Right

Rule: $\angle a + \angle b = 90^\circ$

If $\angle b = 56^\circ$, Then $\angle a = ???$



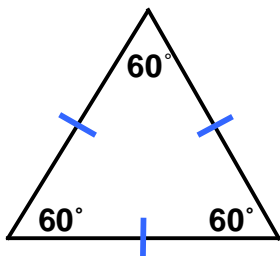
Solution

$$1. \angle a + 56^\circ = 90^\circ$$

$$2. \begin{array}{r} 90^\circ \\ - 56^\circ \\ \hline \angle a = 34^\circ \end{array}$$

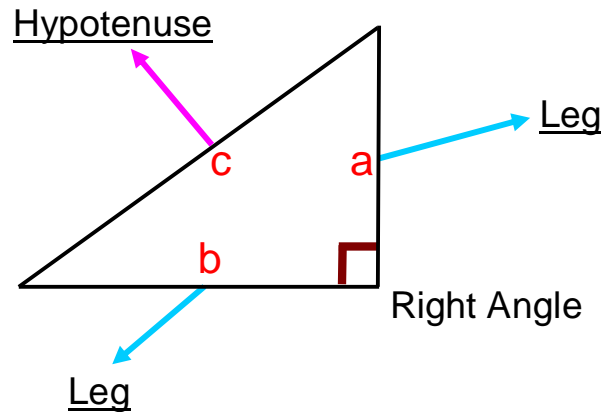
Check: $34^\circ + 56^\circ = 90^\circ$

Equilateral



Rule: Each angle = 60°

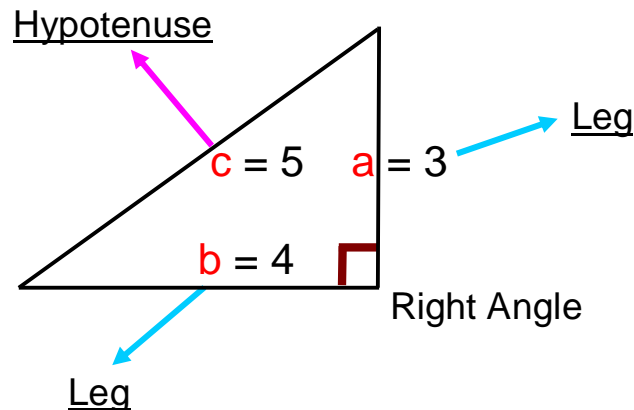
The 3-4-5 Right Triangle



For the sides of every Right Triangle:

$$a^2 + b^2 = c^2$$

Important—the **sides** of the **3-4-5** Right Triangle are **Whole Numbers**.



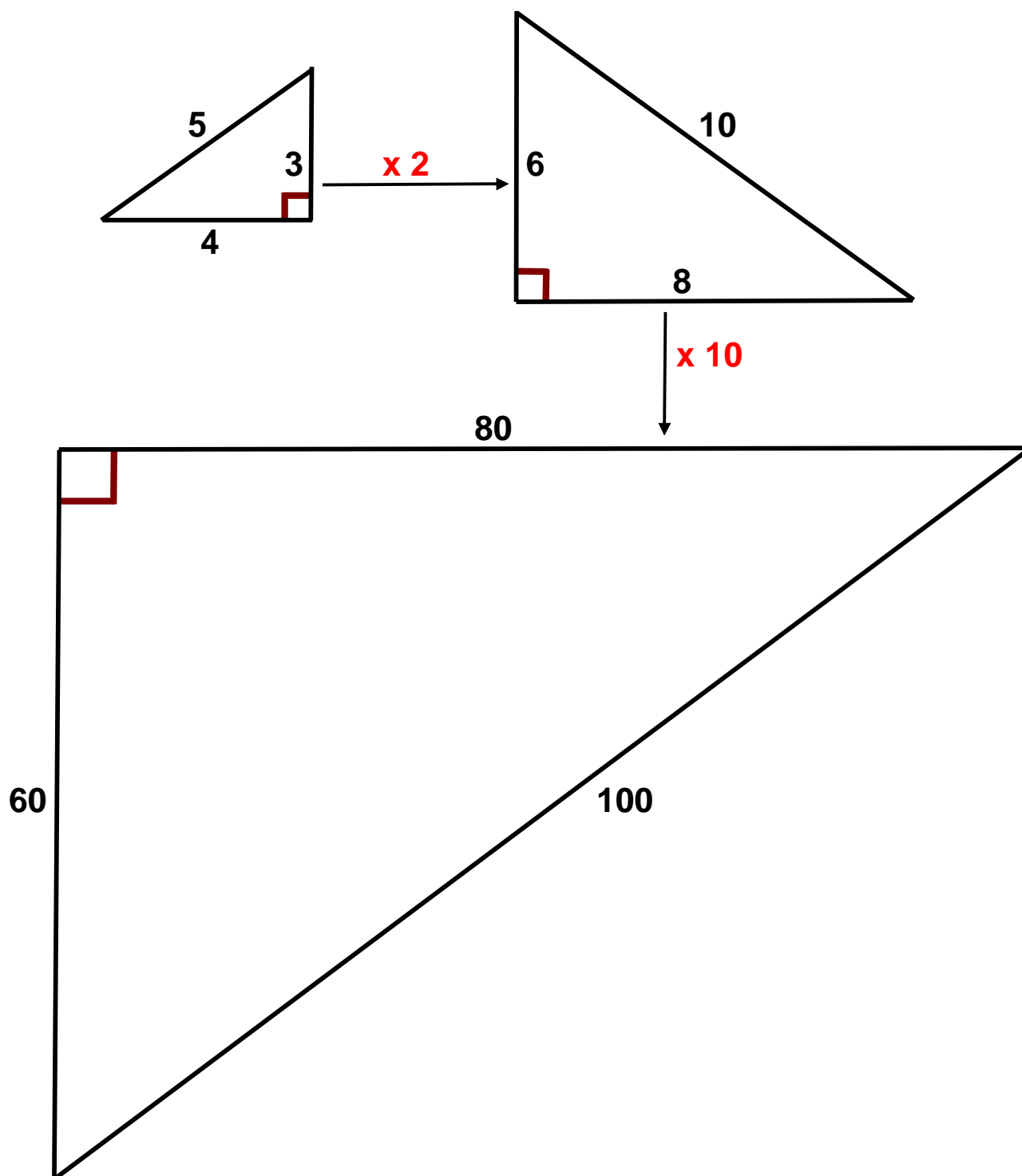
So:

$$\begin{array}{r}
 3^2 + 4^2 = 5^2 \\
 \downarrow \quad \downarrow \quad \downarrow \\
 9 + 16 = 25 \\
 \underbrace{\hspace{2cm}} \\
 25 = 25
 \end{array}$$

Remember the 3-4-5 because you will see it on the GED test!

Multiples of the 3-4-5 Right Triangle

Multiples can be any number—2 or maybe 10—but it's still the 3-4-5 underneath.

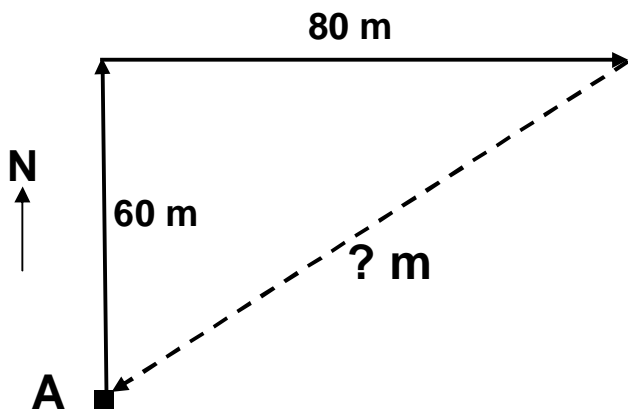


Remember these – you will see them on the GED test!

Recognize the 3-4-5 Right Triangle

Typical GED Question

A ship departs from Point A, and sails 60 miles due north. Then it turns due east and sails for 80 miles. How many miles must the ship sail to return directly to Point A?



First of all, the turn from north to east is a right angle. (Did you know that ?)

The Long Solution

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 60^2 + 80^2 &= c^2 \\
 3600 + 6400 &= c^2 \\
 \underbrace{\hspace{1.5cm}} & \\
 c^2 &= 10,000 \\
 c &= \sqrt{10,000} = 100 \text{ (Answer)}
 \end{aligned}$$

A Faster Solution

Use a multiple of 3-4-5!

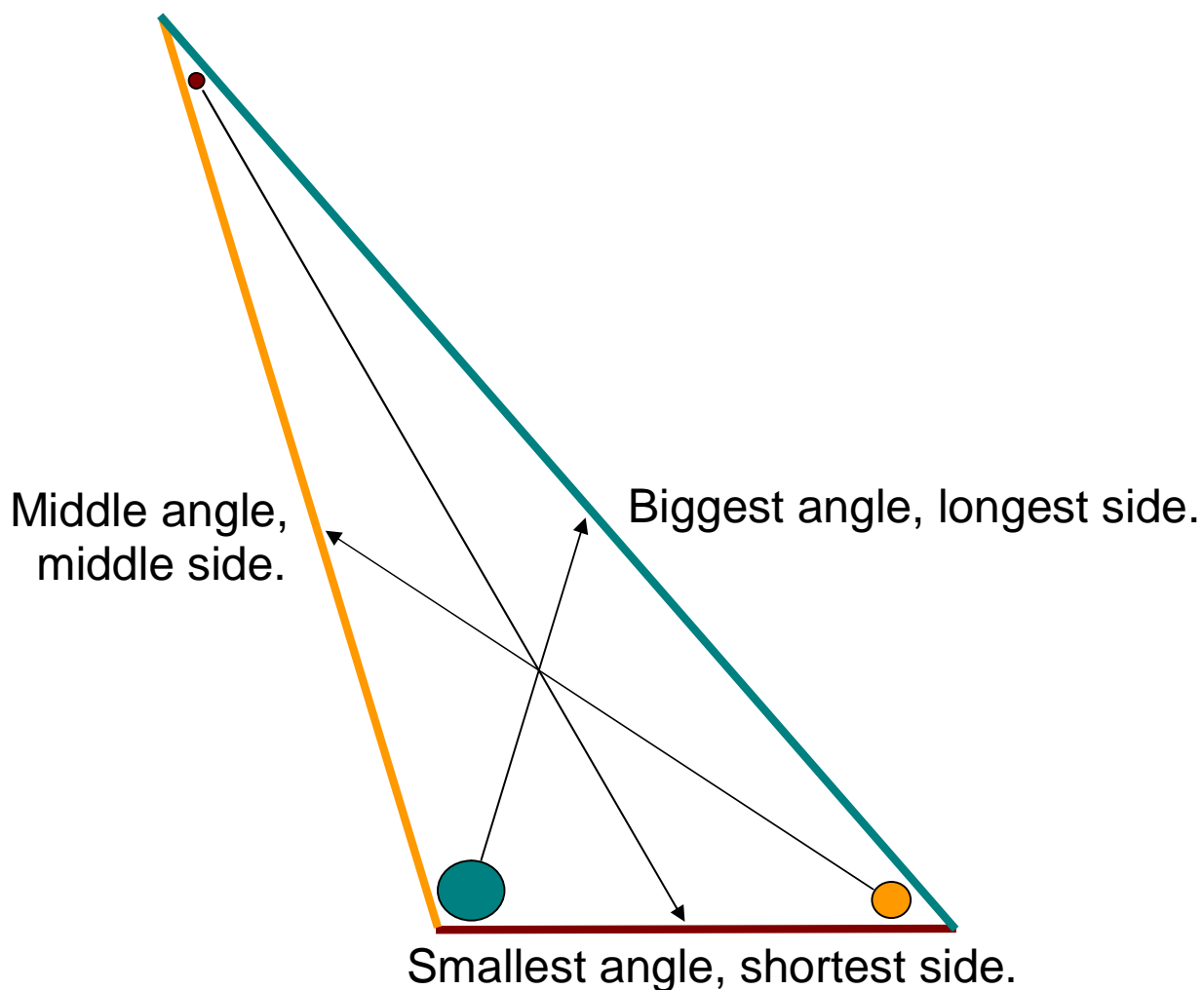
$$6-8-10 \times 10 = 60-80-100 \text{ (Answer)}$$

This easy shortcut saves precious time on the test! Do you use it?

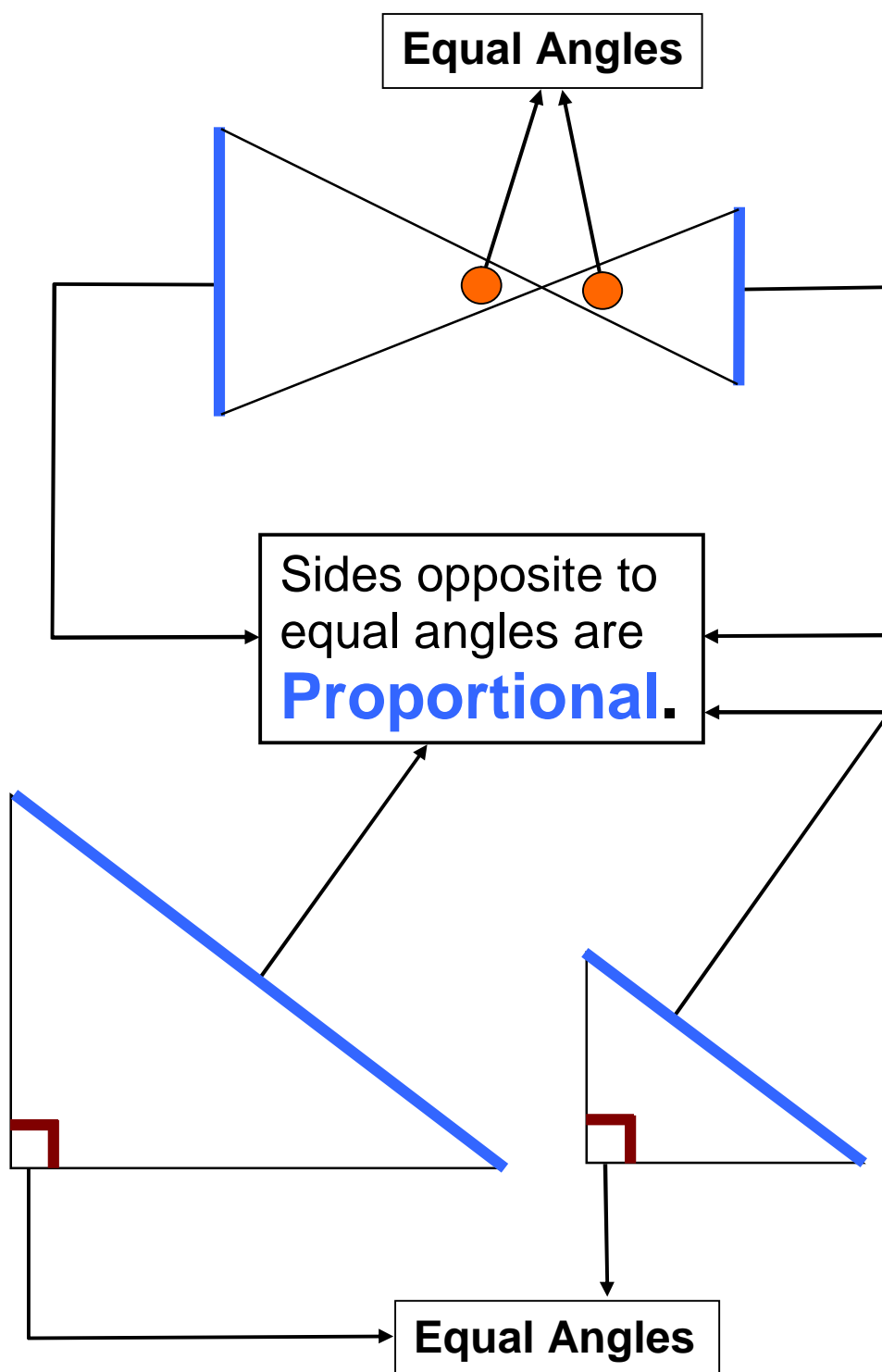
Angles and Opposite Sides

Rule:

The bigger the angle, the longer the opposite side.

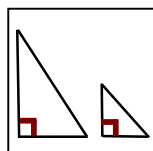


Similar Triangles: Equal Angles, Corresponding Sides

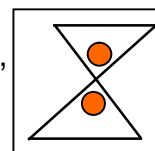


To Solve Similar Triangles: Mark Corresponding Sides

1. Mark the sides opposite to the equal angles



or,

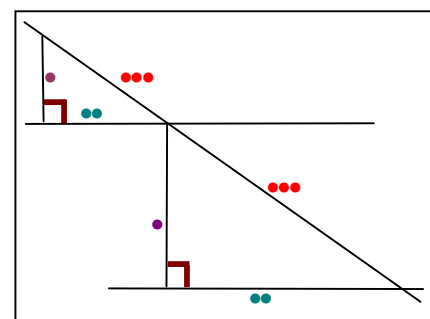
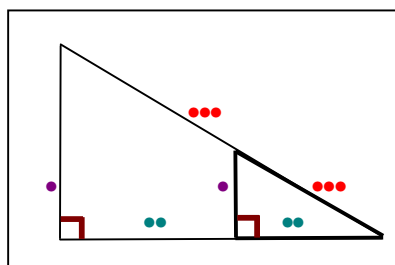
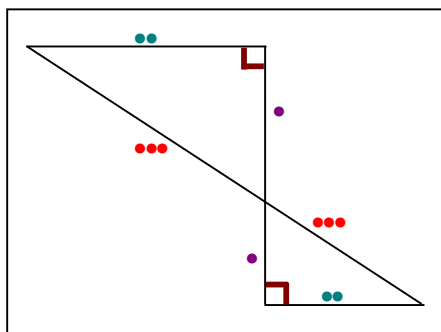


like this: ●●●

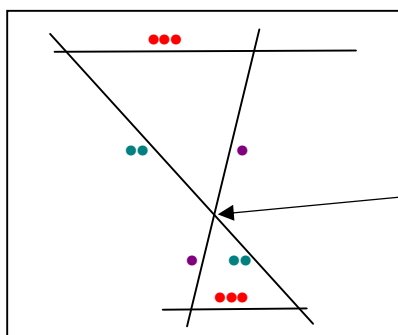
2. Look at the remaining sides, mark the **longer pair** of lines like this: ●●

3. Mark the shorter **remaining pair** of lines like this: ●

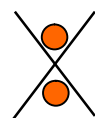
Usually like this:



Sometimes like this:

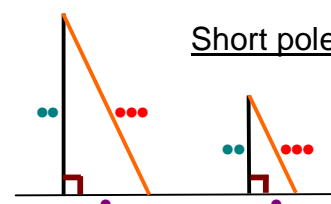


Equal Angles



● Sun

Tall pole



Short pole

Now you have 3 pairs of corresponding sides—use them in proportions.

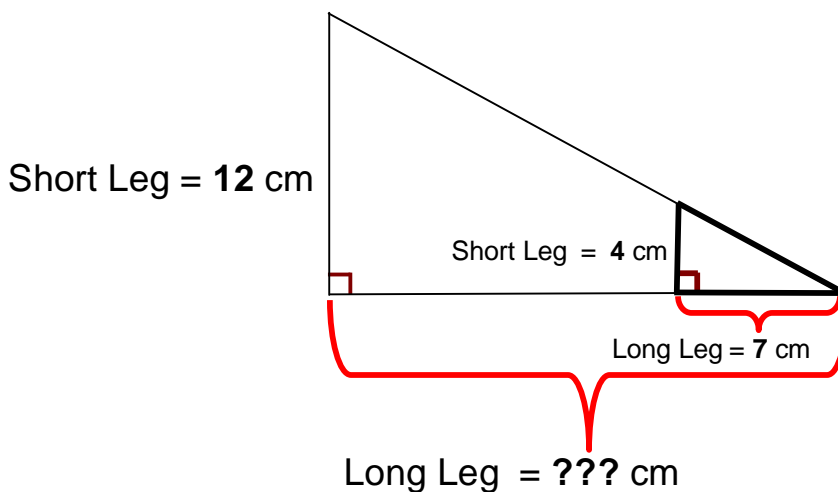
Similar Triangles on the GED



Proportions again! $\frac{\text{red}}{\text{orange}} = \frac{\text{red}}{\text{orange}}$

Typical GED Question

Here are two similar right triangles: (same shape, and equal angles).



Question:

What is the length of the long leg of the big triangle?

Solution:

Corresponding sides—opposite to equal angles—are your “apples” and “oranges.”

$$\frac{\text{red}}{\text{orange}} = \frac{\text{red}}{\text{orange}}$$

$\frac{4 \text{ cm (Short Leg)}}{7 \text{ cm (Long Leg)}} = \frac{12 \text{ cm (Short Leg)}}{n \text{ cm (Long Leg)}}$

$$n = \frac{7 \text{ cm} \times \cancel{12 \text{ cm}}}{\cancel{4 \text{ cm}}} = 21 \text{ cm}$$

Units in Geometry

One Dimension:

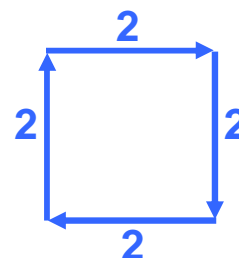
Length

$$L = 2 \text{ ft}$$



Perimeter

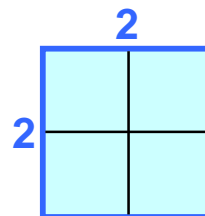
$$P = 2 \text{ ft} + 2 \text{ ft} + 2 \text{ ft} + 2 \text{ ft} = 8 \text{ ft}$$



Two Dimensions:

Area:

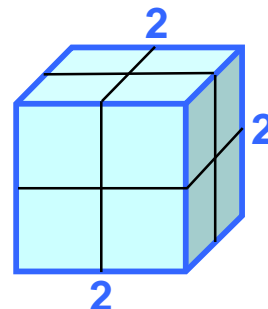
$$A = 2 \text{ ft} \times 2 \text{ ft} = 4 \text{ ft}^2$$



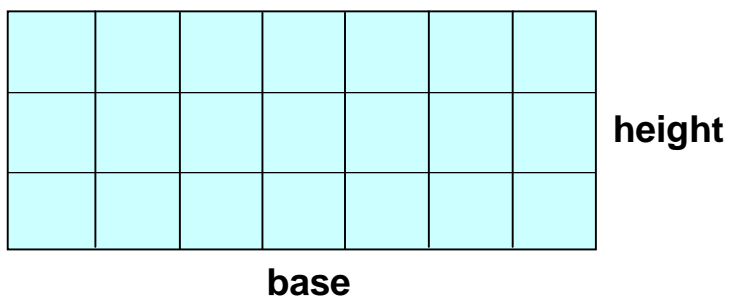
Three Dimensions:

Volume:

$$V = 2 \text{ ft} \times 2 \text{ ft} \times 2 \text{ ft} = 8 \text{ ft}^3$$

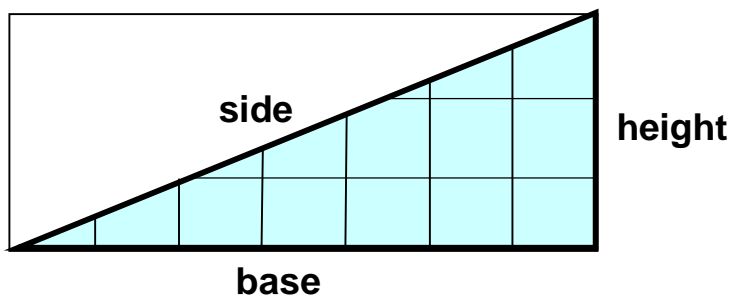


Perimeter and Area: Rectangles and Triangles



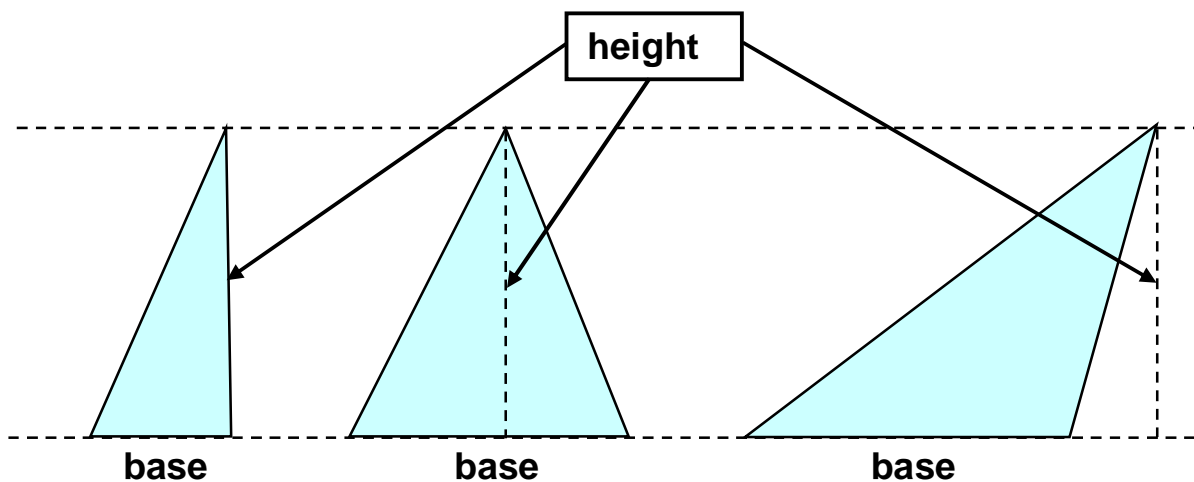
$$P = b + b + h + h$$

$$A = b \times h$$



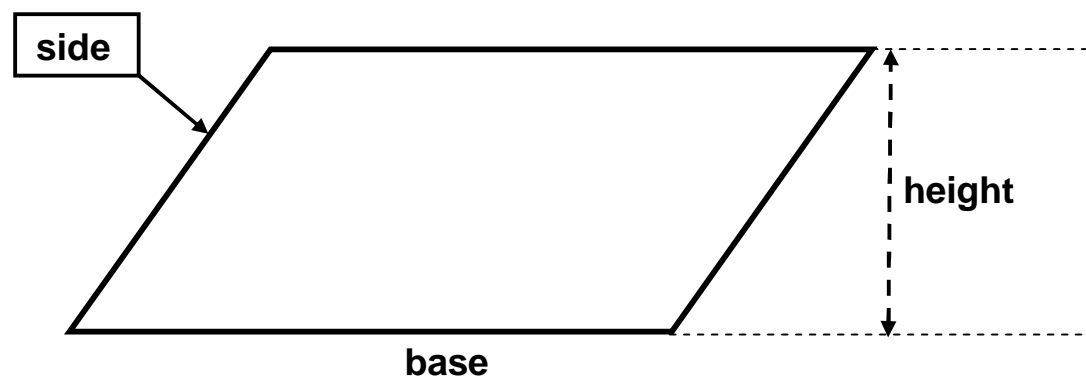
$$P = s + h + b$$

$$A = \frac{1}{2} \times b \times h$$



$$A = \frac{1}{2} \times b \times h$$

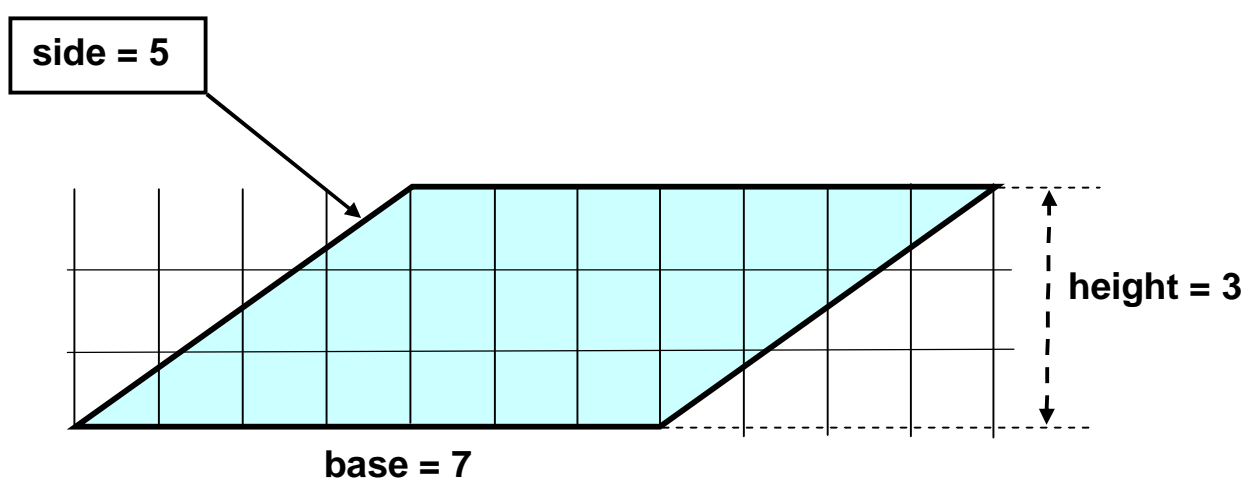
Perimeter and Area: Parallelograms



$$P = b + b + h + h$$

$$A = b \times h$$

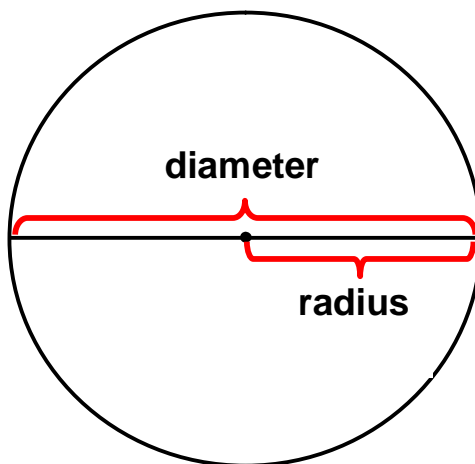
Example:



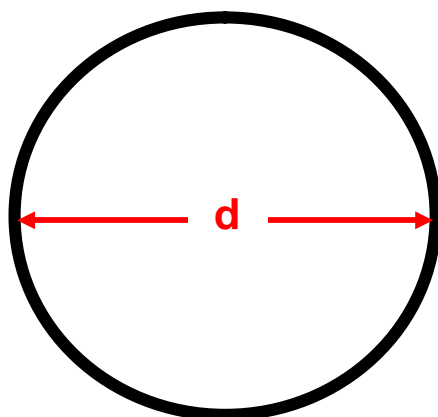
$$P = 7 + 7 + 5 + 5 = 28$$

$$A = 7 \times 3 = 21$$

Circumference and Area: Circles



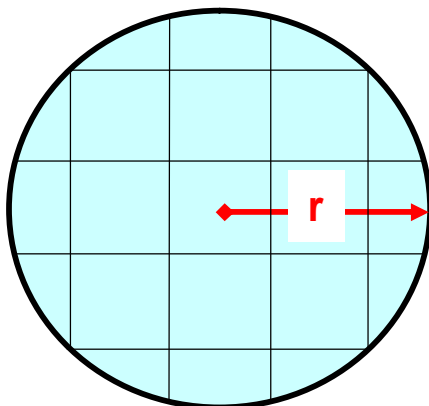
Circumference: The distance around the edge.



$$C = \pi D$$

$$\pi = 3.14$$

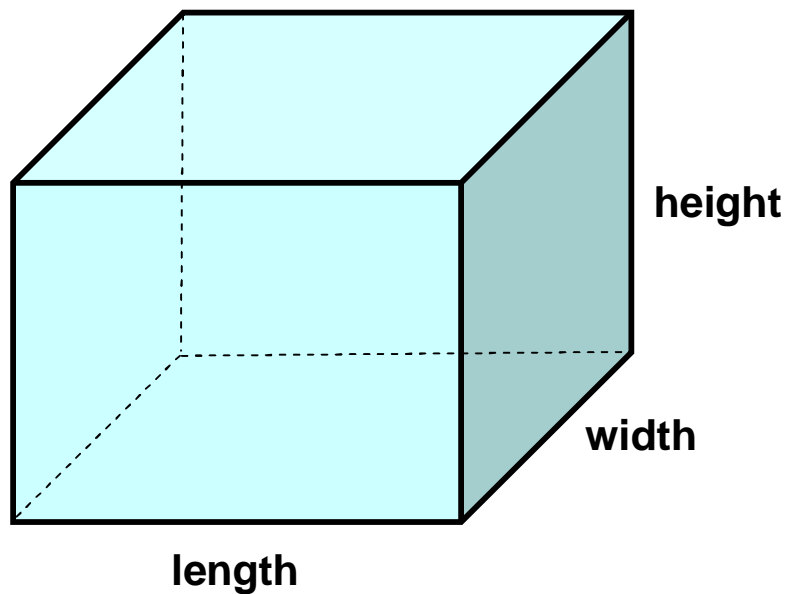
Area: The number of squares in the middle.



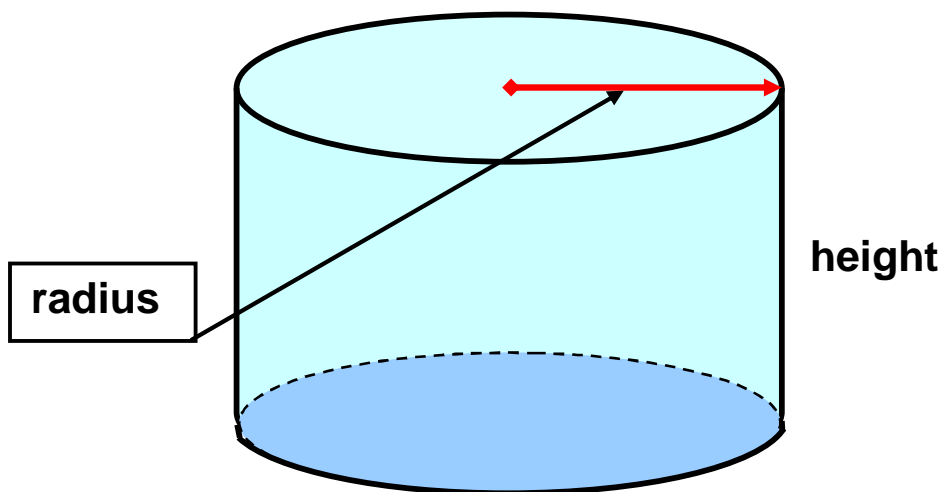
$$A = \pi r^2$$

$$\pi = 3.14$$

Volume



$$V = \underbrace{\text{length} \times \text{width}}_{\text{area of the base}} \times \text{height}$$



$$V = \underbrace{\pi r^2}_{\text{area of the base}} \times \text{height}$$