

GED Math Graphics: Illustrated Guide to Self-Confidence

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Unit 4

ALGEBRA

- Basic Algebra
- Formulas
- Coordinate Grids
- Algebra Problems

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*** Typical GED Questions

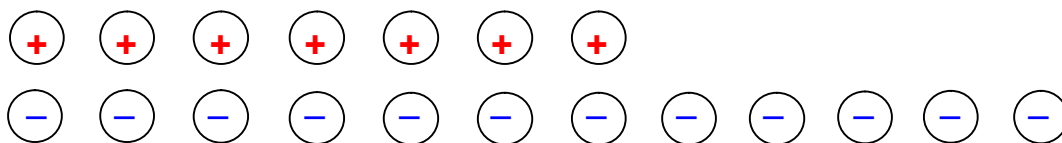
Counting with Bottle Caps

Of course you know this: $12 - 7 = 5$

- **But to be ready for algebra, you also have to know this: $7 - 12 = ?$**

Another Way of Counting

- You can use white plastic bottle caps.
- Mark some with a “+” and some with a “-”.
- Then line up 7 “+” caps and 12 “-” caps, like the diagram:

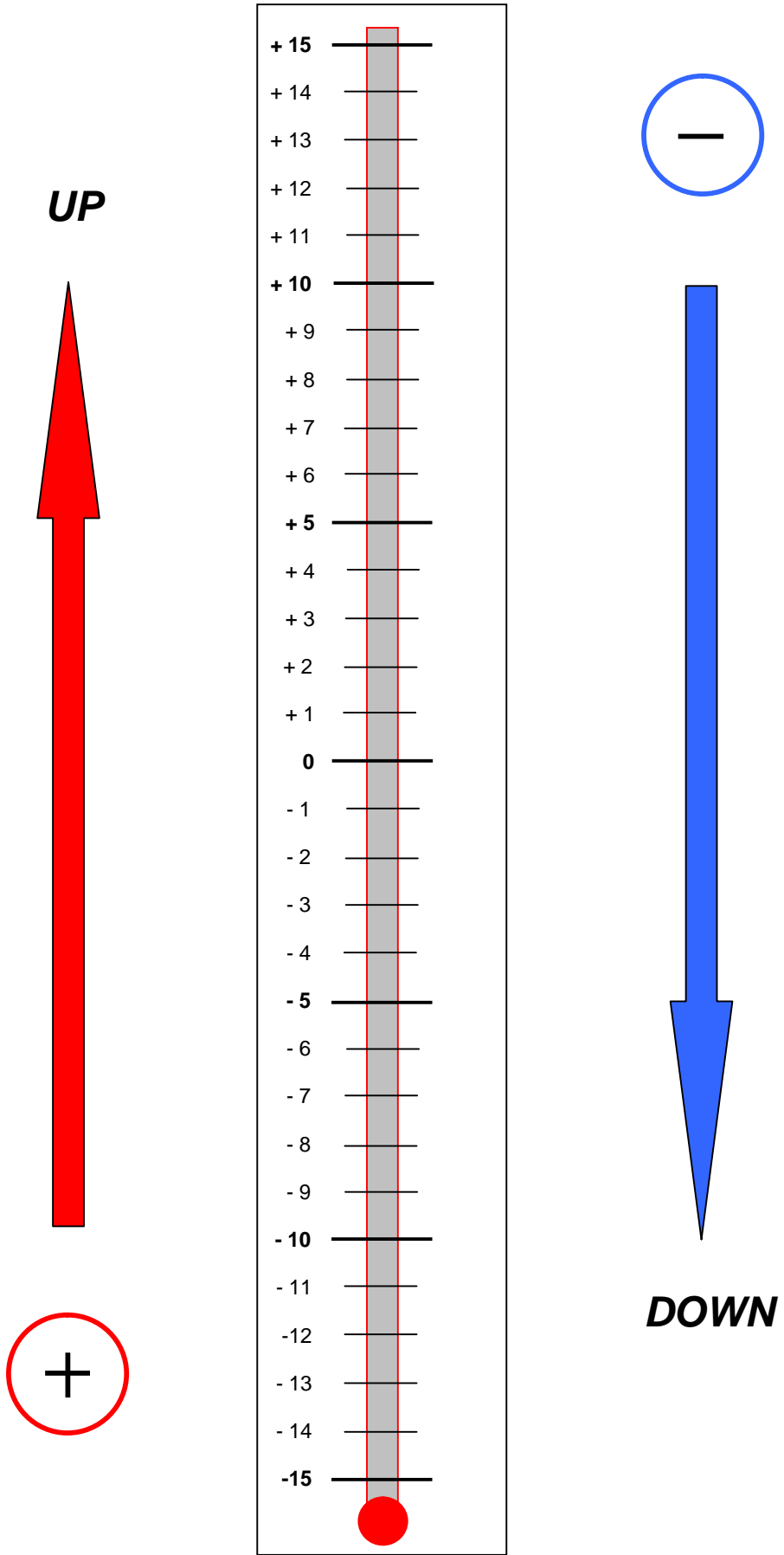


- Since $+1 - 1 = 0$, remove the seven pairs of “+” and “-” caps, on the left.
- There are 5 “-” caps, remaining on the right. **So the answer is - 5.**

And Another Way – The Thermometer

- The Thermometer (next page) is a number line. For $7 - 12$, start on **+ 7**.
- Count down **- 12** points, and again you find that **the answer is - 5**.

The Thermometer



To Multiply and Divide Signed Numbers

Rule

- Same Sign, Positive

$$\begin{array}{cccccc}
 + & \bullet & + & = & + \\
 - & \bullet & - & = & +
 \end{array}$$

- Different Signs, Negative

$$\begin{array}{cccccc}
 + & \bullet & - & = & - \\
 - & \bullet & + & = & -
 \end{array}$$

Example:

$$-(-5) + (-7) - (3) + (8) = ?$$

1. First, Multiply or Divide

$$\begin{array}{cccc}
 \downarrow & \downarrow & \downarrow & \downarrow \\
 +5 & -7 & -3 & +8
 \end{array}$$

2. Next, Add or Subtract

$$\text{a.) } +5 + 8 = +13$$

$$\text{b.) } -7 - 3 = \underline{-10}$$

$$\text{c.) } \quad \quad \quad +3$$

Algebraic Expressions

Simplify:

To collect like terms, remember the thermometer.

Example:

$$9x \quad -8 \quad -5x \quad +9 \quad = \quad ???$$

Step 1

$$9x \quad -8 \quad -5x \quad +9 \quad = \quad 4x$$

Step 2

$$-8 \quad +9 \quad = \quad +1$$

Step 3

$$4x \quad +1$$

Expand:

Example: (Look back at Unit 1, p. 11)

Step 1

$$3(5x - 4) \quad (3 \cdot 5x) = 15x$$

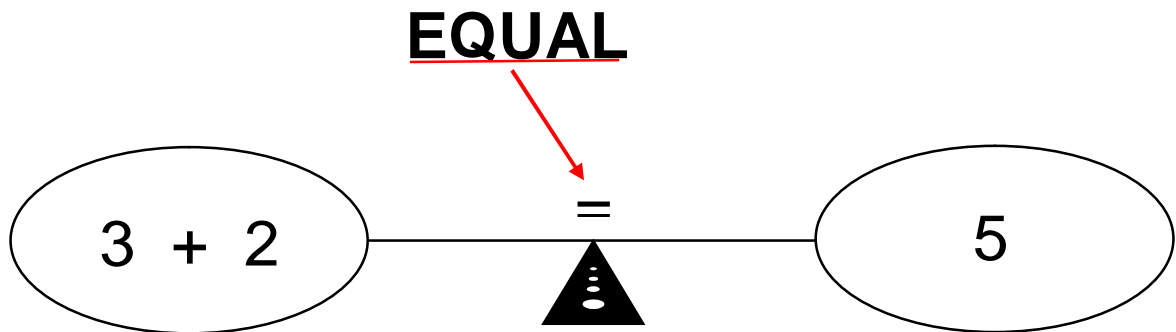
Step 2

$$3(5x - 4) \quad (3 \cdot -4) = -12$$

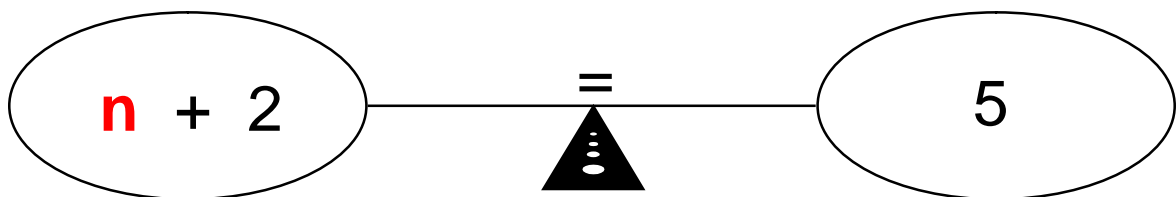
Step 3 (Answer)

$$15x \quad -12$$

Equation: A Balance of Two Equal Sides

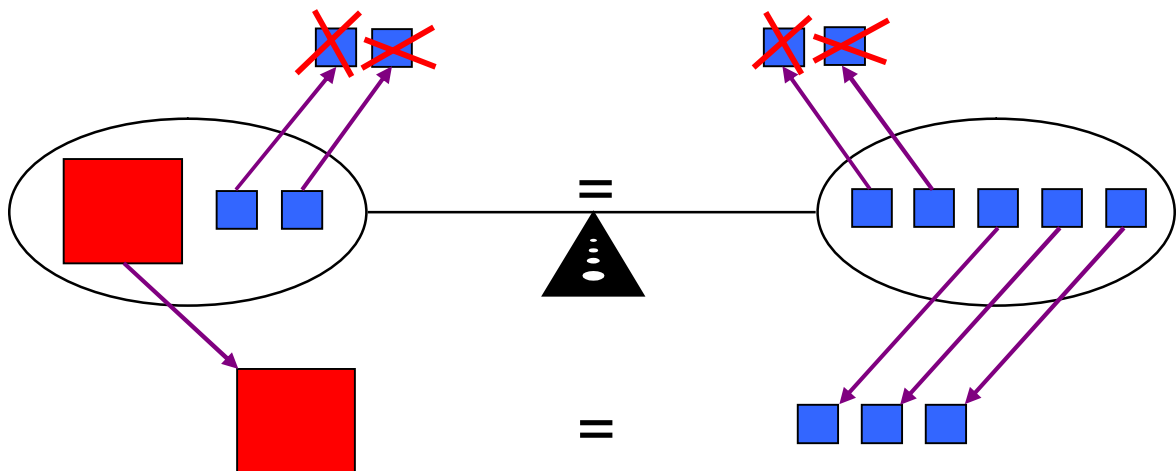


With a **Variable**:



To Solve:

1. Get **n** by itself.
2. Keep sides equal.



Get **n** by Itself – Equations With One **n**

1. Move number terms away from **n**.

a. Subtract the positive.

$$\begin{array}{r}
 \mathbf{n} + 2 = 3 \\
 \phantom{\mathbf{n}} \downarrow \\
 \mathbf{n} + 2 = 3 \\
 \quad \underline{-2} \quad \underline{-2} \\
 \mathbf{n} = 1
 \end{array}$$

b. Add the negative.

$$\begin{array}{r}
 \mathbf{n} - 2 = 3 \\
 \phantom{\mathbf{n}} \downarrow \\
 \mathbf{n} - 2 = 3 \\
 \quad \underline{+2} \quad \underline{+2} \\
 \mathbf{n} = 5
 \end{array}$$

Get **n** by Itself – When **n** Has a Number

2. Divide or multiply to get one **n**.

a. Divide the multiplier.

$$2 \mathbf{n} = 6$$

$$\begin{array}{c} \downarrow \\ \frac{\cancel{2} \mathbf{n}}{\cancel{2}} = \frac{\cancel{6}}{\cancel{2}} \end{array}$$

$$\mathbf{n} = 3$$

b. Multiply the divisor.

$$\frac{\mathbf{n}}{2} = 3$$

$$\begin{array}{c} \swarrow \\ \cancel{2} \times \frac{\mathbf{n}}{\cancel{2}} = 3 \times 2 \end{array}$$

$$\mathbf{n} = 6$$

To Solve Equations

Example:

$$6n + 2 - 2n - 3 = 5n + 8 - 3n - 3$$

$$4n - 1 = 2n + 5$$

1. Simplify each side.

2. Move n terms to the left.

$$\begin{aligned} 4n - 1 &= 2n + 5 \\ -2n & \\ \hline 2n - 1 &= + 5 \end{aligned}$$

3. Move number terms to the right.

$$\begin{aligned} 2n - 1 &= +5 \\ +1 & \\ \hline 2n &= +6 \end{aligned}$$

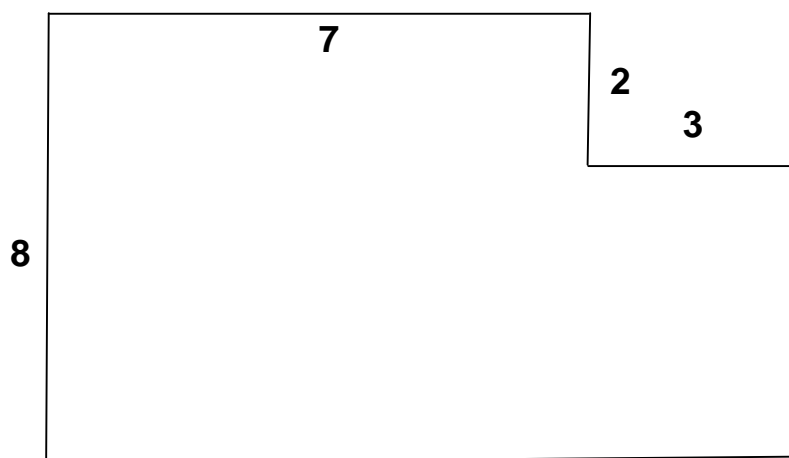
4. Divide or multiply to get one n .

$$\begin{aligned} 2n &= 6 \\ \frac{2n}{2} &= \frac{6}{2} \\ n &= 3 \end{aligned}$$

Formulas

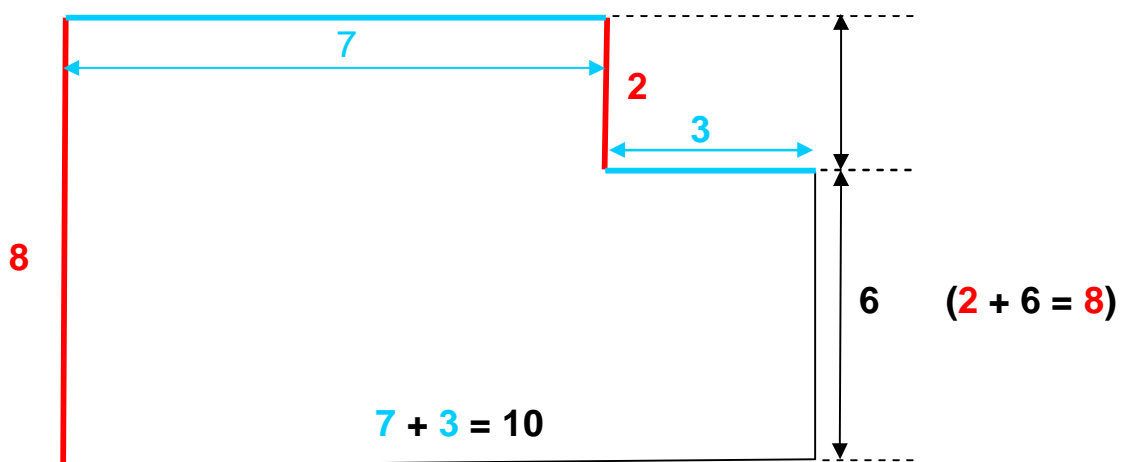
GED Skill – Perimeter

Find all sides and the perimeter of the rectangular figure.



To Solve:

Find each side and then total them.



The Long Solution: Start at the top and go clockwise.

$$P = 7 + 2 + 3 + 6 + 10 + 8 = 36$$

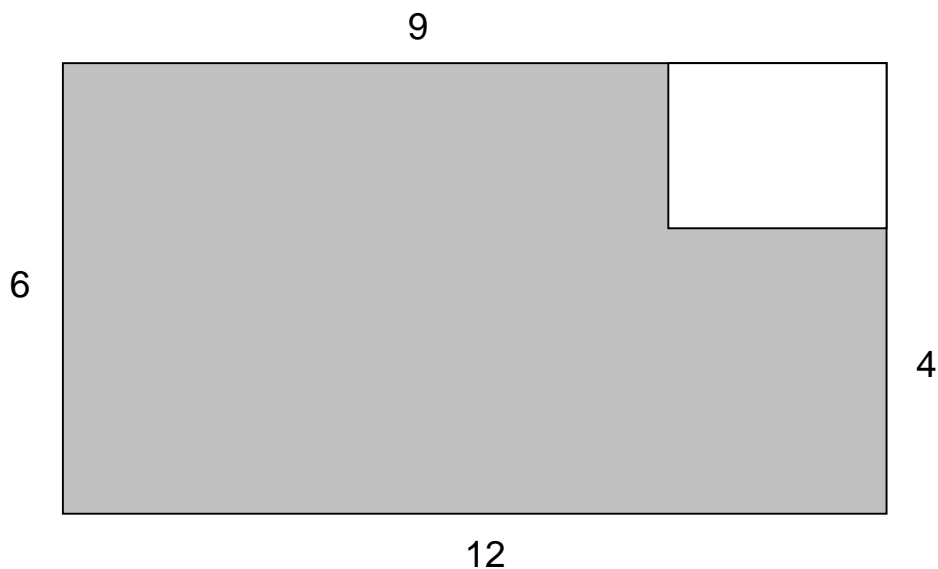
A Faster Solution:

$$P = 10 + 10 + 8 + 8 = 36$$

Formulas

Typical GED Question – Subtract the Smaller Area #1

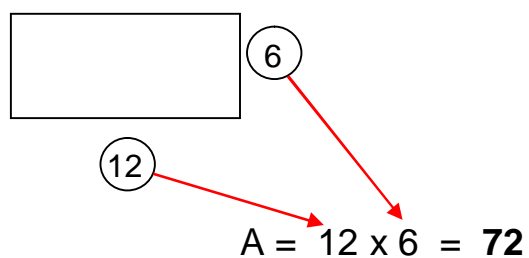
Find the area of the shaded portion of the rectangle.



Step 1

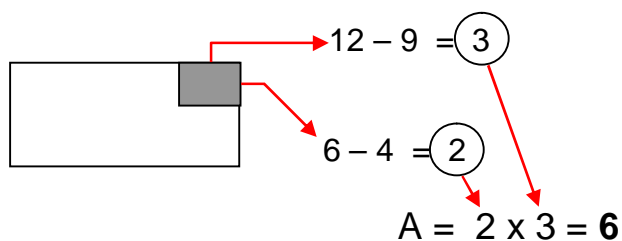
Start with the **large area**.

Area = length x width



Step 2

Find the **small area**.



Step 3

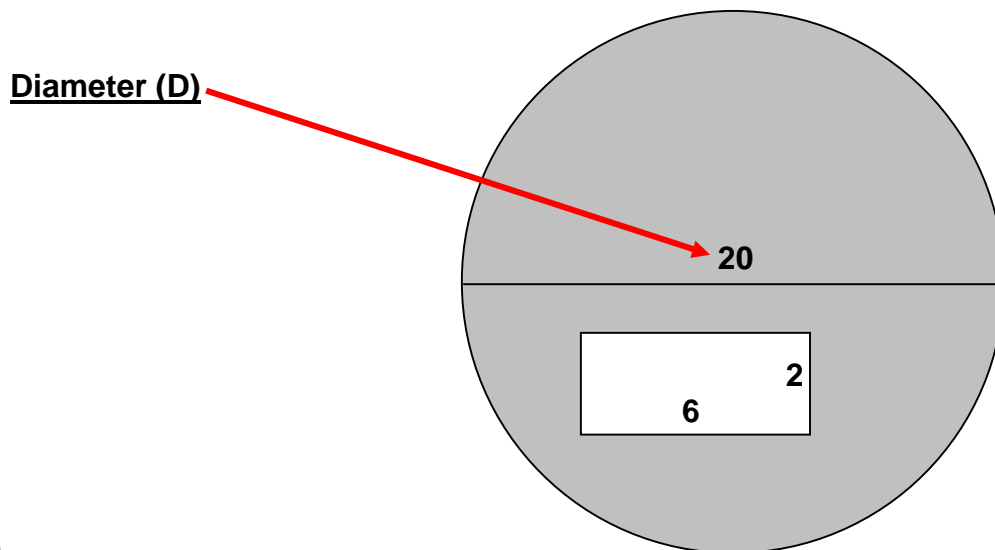
Subtract:

$$72 - 6 = 66$$

Formulas

Typical GED Question – Subtract the Smaller Area #2

Find the area of the shaded portion of the figure.



Step 1

Start with the **large area**.

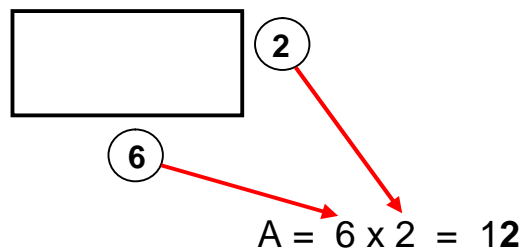
Area = πr^2 **Remember:** the radius is one half the diameter.

$$A = \pi (r)(r) = 3.14 (10)(10) = \mathbf{314}$$

Step 2

Find the **small area**.

Area = length x width



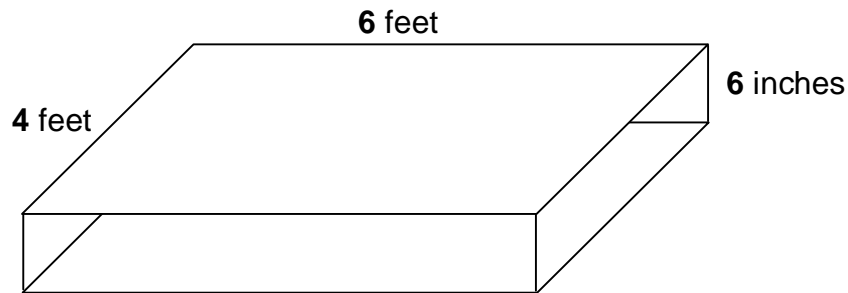
Step 3

Subtract: $314 - 12 = \mathbf{302}$

Formulas

Typical GED Question – Volume with Consistent Units

Find the volume in cubic feet of this rectangular solid.



Volume = Length x Width x Height

BUT if you write this, it's **wrong!**

$$V = 6 \text{ feet} \times 4 \text{ feet} \times 6 \text{ inches}$$

Why? Because all units must be the same. You must change inches to feet.

Because there are 12 inches per foot, the correct formula looks like this:

$$V = 6 \text{ feet} \times 4 \text{ feet} \times \frac{6}{12} \text{ feet}$$

To Solve:

$$V = \cancel{6} \text{ feet} \times \cancel{4} \text{ feet} \times \frac{\cancel{6}}{12} \text{ feet} = 12 \text{ ft}^3$$

$\begin{matrix} 2 \times 2 \\ \cancel{6} \times 2 \end{matrix}$

Formulas

Typical GED Question – Interest with Consistent Units

Find the interest on a loan of \$5500 at a rate of 8% for 9 months.

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time}$$

BUT if you write this, it's **wrong!**

$$I = \$5500 \times 8\% \times 9 \text{ months}$$

Why? Because all units must be the same. You must change months to years.

Because there are 12 months per year, the correct formula looks like this:

$$I = \$5500 \times 8\% \times \frac{9}{12} \text{ year}$$

To Solve with a Calculator:

$$\begin{aligned} I &= \$5500 \times .08 \times (9 \div 12) \\ &= \$5500 \times .08 \times .75 = \mathbf{\$330.00} \end{aligned}$$

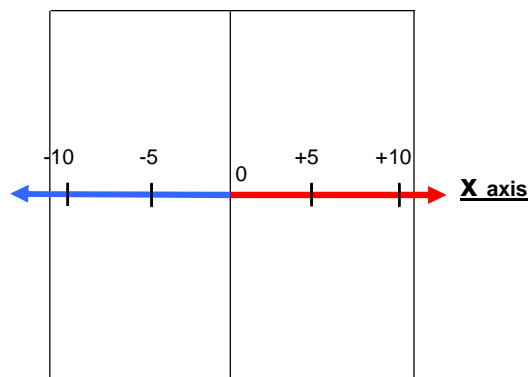
To Solve by Cancelling:

$$I = \cancel{\$5500} \times \frac{\cancel{8}^{4 \times 2}}{\cancel{100}^{3 \times 3}} \times \frac{\cancel{9}^{4 \times 3}}{\cancel{12}^{4 \times 3}} = \$55 \times 2 \times 3 = \mathbf{\$330.00}$$

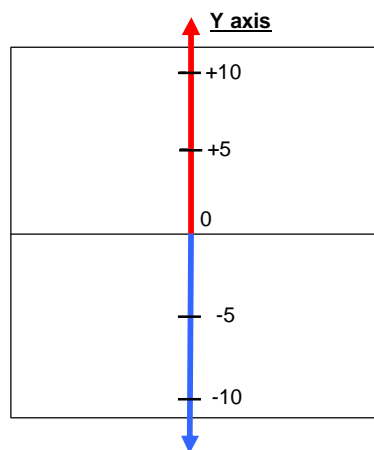
Coordinate Grids

A coordinate grid has two number lines through the middle.

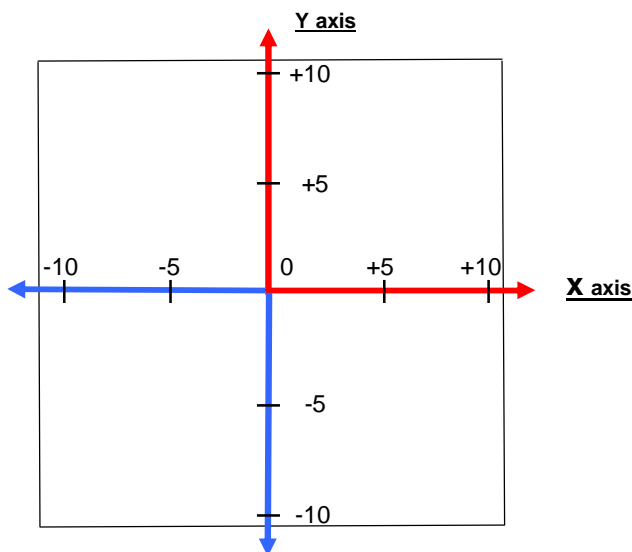
The horizontal line is called the X axis.
It is just like our thermometer, turned sideways.



The vertical line is called the Y axis.
It works just like our thermometer.



Here is a complete coordinate grid, with both thermometers together:

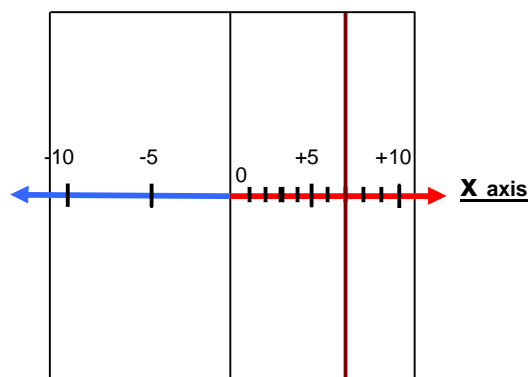


Typical GED Question – Find a Point on a Coordinate Grid

Find the point $(7,-3)$ on a coordinate grid.

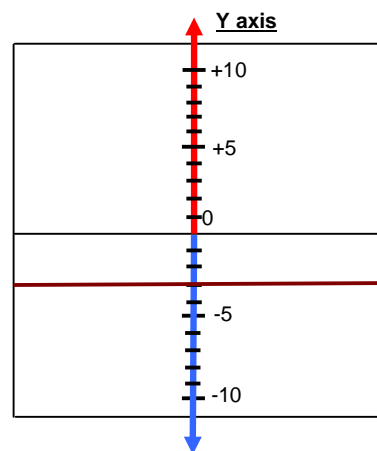
Step 1

The 7 — first number in $(7,-3)$ — is the **X** coordinate.
Find $X = 7$. Draw a vertical line, where **X** always = 7.



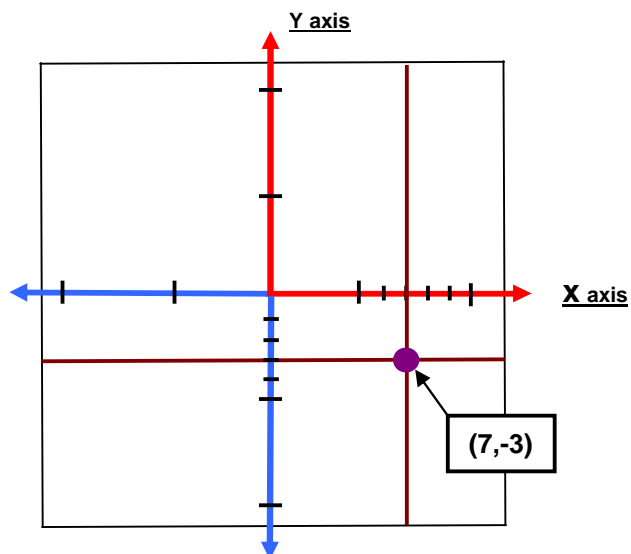
Step 2

The -3 — second number in $(7,-3)$ — is the **Y** coordinate.
Find $Y = -3$. Draw a horizontal line, where **Y** always = -3.



Step 3

Mark the point $(7,-3)$ where the lines cross.
This is the only point where $X = 7$, **and** $Y = -3$.



Slope – How Steep is the Climb?

Easy:

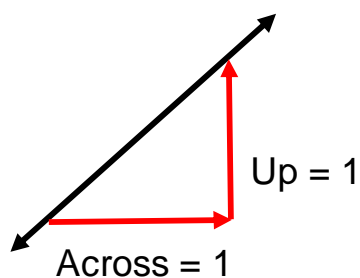
<1



$$\text{Slope} = \frac{1}{3}$$

Medium:

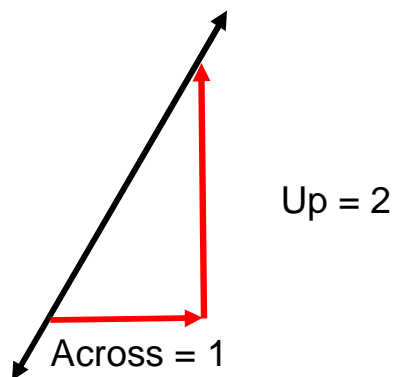
$=1$



$$\text{Slope} = \frac{1}{1} = 1$$

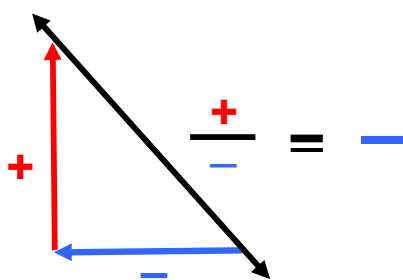
Steep:

>1

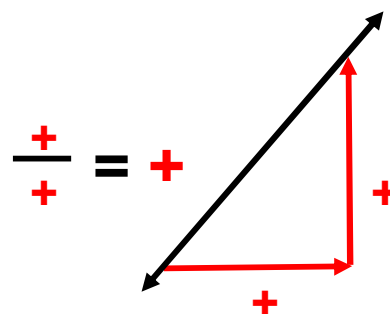


$$\text{Slope} = \frac{2}{1} = 2$$

Leans Left: Negative

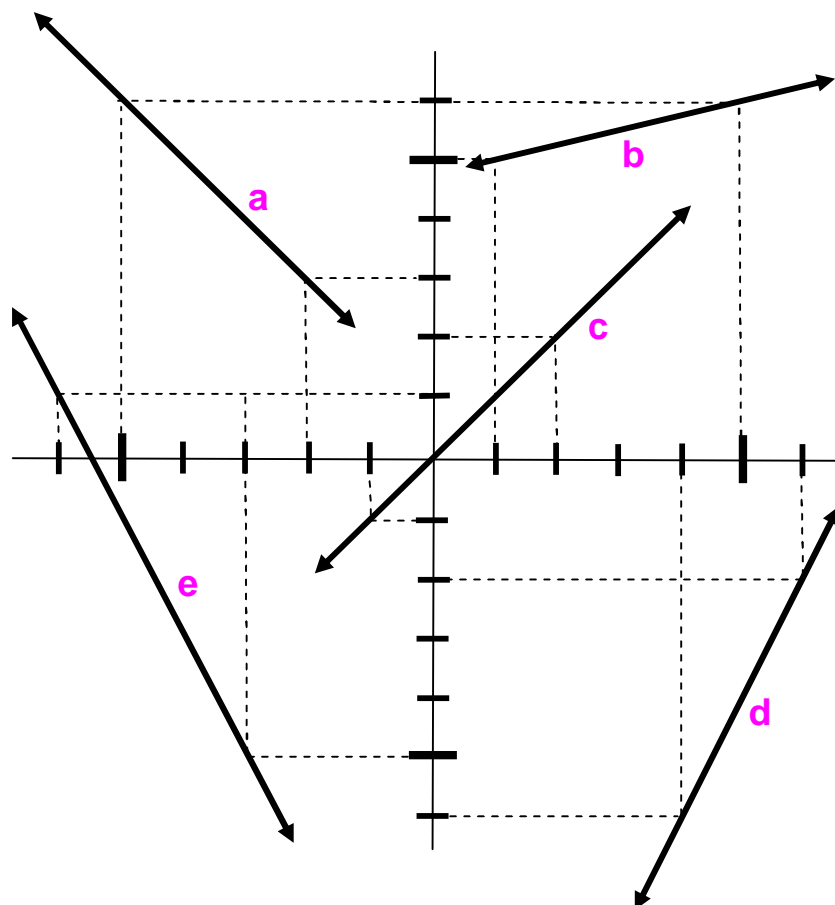


Leans Right: Positive



Slopes – Smart Guesses

Typical GED Question – Which is the Best Guess for Each Slope?



Choices:

- 1.) + 1
- 2.) +0.25
- 3.) - 2
- 4.) + 2
- 5.) - 1

Solutions:

- (a) and (e) lean to the left; they are **negative**.
(e) is steeper than (a); its value is greater.
So the **slope** of (e) = **-2**, and (a) = **-1**.
- (b), (c) and (d) lean to the right; they are **positive**.
- (b) is an easy climb. Its **slope is <1**, so the correct choice must be **+0.25**.
- (c) is a medium climb. The correct choice must be **+1**
- (d) is a steep climb. Its **slope is >1**, so the correct choice must be **+2**.

Typical GED Question – Find a Point in an Equation

For the equation:

$$2y = 3x + 4$$

- 1.) What are the coordinates of the point when $x = 2$?

Solution:

$$2y = (3)(2) + 4$$

$$2y = 6 + 4$$

$$\begin{array}{r} \cancel{2} \times 5 \\ \cancel{2}y = \cancel{10} \\ \cancel{2} \quad \quad \cancel{2} \end{array}$$

$$y = 5$$

Answer: (2,5)

- 2.) What is the y-intercept?

Solution:

(Clue: At the y-intercept, $x = 0$.)

$$2y = (3)(0) + 4$$

$$2y = 0 + 4$$

$$\begin{array}{r} \cancel{2} \times 2 \\ \cancel{2}y = \cancel{4} \\ \cancel{2} \quad \quad \cancel{2} \end{array}$$

$$y = 2$$

Answer: (0,2)

Think in Algebra

<u>Words</u>	<u>Algebra</u>
A number	x
4 more than a number	$x + 4$
3 times a number	$3x$
2 less than 6 times a number	$6x - 2$
A number divided by 4	$\frac{x}{4}$
Two consecutive even numbers	$x, x + 2$
Three consecutive odd numbers	$x, x + 2, x + 4$

Solve Algebra Puzzles

Typical GED Questions

- The sum of what three consecutive even numbers is 42?

Solution: $x + (x + 2) + (x + 4) = 42$

$$3x + 6 = 42$$

$$3x = 36$$

Answer $x = 12, x + 2 = 14, x + 4 = 16$

Check: $12 + 14 + 16 = 42$

- Joe's brother is 3 years less than twice Joe's age. Together their ages total 24. What the age of Joe's brother?

Solution: Joe's age = x

His brother's age = $2x - 3$

$$x + (2x - 3) = 24$$

$$3x - 3 = 24$$

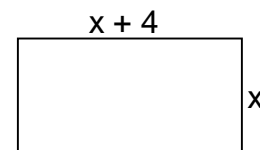
$$x = 9 \quad (\text{Answer? NO!})$$

His brother's age = $2x - 3$

Answer $2x - 3 = 2(9) - 3 = 15$

Check: $15 + 9 = 24$

- The perimeter of this rectangle is 32. What is its length?



Solution: $P = (x + 4) + (x + 4) + x + x$

$$P = 4x + 8$$

$$4x + 8 = 32$$

$$4x = 24$$

$$x = 6 \quad (\text{Answer? NO!})$$

Length = $x + 4$

Answer $6 + 4 = 10$

Check: $P = 10 + 10 + 6 + 6 = 32$